



# FINITE ELEMENT SIMULATION OF PUNCH THROUGH TEST USING A CONTINUOUS SURFACE CAP MODEL

Aniket Patil<sup>1,2</sup>, Bjørnar Sand<sup>1</sup>, Lennart Fransson<sup>2</sup>

<sup>1</sup>Northern Research Institute, Narvik, NORWAY

<sup>2</sup>Luleå University of Technology, Luleå, SWEDEN.

## ABSTRACT

An attempt has been made to calibrate the material model parameters of the continuous surface cap model with data from punch through tests performed in the Northern Gulf of Bothnia. An axisymmetric finite element model has been used to simulate the field tests. The continuous surface cap model based on a combination of elastic-plastic and continuum damage mechanics formulation is used as constitutive model for ice rubble. Material properties such as internal friction angle, cohesion and Young's modulus are evaluated in a parametric study and the response is compared to the experimental data for the chosen test. An optimization algorithm is used for determining the parameters used for describing the continuous surface cap model. The material model parameters are chosen to get best fit to test load displacement curve. Conclusion has been drawn based on the application of continuous surface cap model on ice rubble.

## INTRODUCTION

Recent development of exploration and production of hydrocarbon has boosted research activities in arctic waters. To determine the design load levels of arctic offshore structures, it is important to know the strength of first year ridges. Sea ice ridges are common ice features in arctic and subarctic seas. Sea ice ridges form either by ice floes compression or by shearing each other. In ice ridge formation, ice blocks pile up below and above water line. Sea currents and wind are main driving forces. Sea ice ridges mainly contain ice pieces. Ice ridges can be further divided into two parts by their position in the ridge. Consolidated part of ridge is above the water line and it is called the consolidated layer. The unconsolidated part of ridges lies below and above consolidated layer. The part above the consolidated layer is called ridge sail and the one below is called ridge keel. When outside temperature is below the freezing temperature, consolidation starts at waterline and spreads towards to the bottom. Therefore, the consolidated layer has lowest degree of porosity and contains air pockets. As sea ice ridge is porous feature, all parts contain varying degree of porosity. Because of their location with respect to water line, ridge keel contains water and air in its pores, whereas ridge sail contains snow and air in their pockets. Rubble below water line already loaded with hydrostatic pressure due to the surrounding water. Because of hydrostatic equilibrium, volume of sail is approximately one tenth of volume of keel. Ice pieces in keel rubble can be loose block piled together or bonded together with cohesive bonds. Presence of cohesive bond restricts the individual movement of ice blocks. Therefore, it is important to know the contribution of these cohesive bonds in ridge keel load on marine structure. Once the cohesive bonds breaks (if there are any), frictional resistance and crushing strength of ice block gives the remaining of ridge keel load. It is clear that both initial and post failure behaviour of rubble will be affected by aging, thermal conditions, block shape, initial stress conditions of the

rubble and testing strain rate. So we need a material model capable to capture the behaviour of ice rubble deformation and failure at all stages. In this paper, a continuous surface cap model (CSCM) MAT 145 in LS-Dyna is used to simulate ice rubble behaviour in punch through test event. This material model is developed by Schwer and Murray (1994) and implemented by Murray (2007). This model is coupled with continuum damage mechanics formulation to provide strain-softening feature. An axisymmetric 2D finite element model is created with Lagrangian finite element mesh formulation. Detailed description is given in coming sections.

## **MECHANICAL PROPERTIES OF ICE RUBBLE**

Punch through was found to be extremely brittle in a 50 cm ice sheet in the gulf of Bothnia, Fransson (1985). Several laboratory and in-situ tests have been conducted to characterise mechanical properties of rubble in the past. The main focus of these tests was to understand different failure modes of ice rubble under different boundary condition and to estimate ice rubble strength. Several mechanisms have been proposed for ice rubble failure by Timco et al. (2000), Heinonen (2004), Liferov (2005c), Shafrova (2007). Ice rubble can fail by different mechanisms due to the complicated internal structure. As the rubble is loaded with hydrostatic pressure, main failure modes are shear and compaction as platen progresses into rubble Azarnejad and Brown (2001) , Heinonen (2004) and Liferov (2005c). Three different physical processes that can be identified during rubble deformation. They are as follows.

1. Breaking of the freeze bonds between the ice blocks.
2. Movement of the rubble blocks.
3. Failure of the ice blocks.

In other words, the strength and morphology of the freeze bonds, the size, shape, orientation and strength of the rubble blocks are all important for estimating the overall ice ridge strength Heinonen and Määttänen (2000a), Høyland (2002), Høyland (2004), Shafrova (2007). Based on these primary failure modes, several material models have been proposed by Wong et al. (1990), Azarnejad, Frederking et al. (1999), Timco and Cornett (1999), Heinonen (2004), Liferov (2005a) and Serré (2011). Origin of these models is cohesive-frictional model proposed by Mohr-Coulomb. Considering similarities between ice rubble and sand like materials, Mohr-Coulomb proposes that cohesion falls out when plotting against shear strength. Smooth approximation of Mohr-Coulomb is used for better numerical stability proposed by Drucker-Prager. Similarities between cohesive frictional type material like sand or concrete have led to use geological cap models to simulate ice rubble. A cap is added to simulate hardening by compaction of rubble. Dilation or volumetric expansion by shearing has been taken care by choosing combination of friction angle and cohesion. These models are used to simulate pre-peak and peak behaviour of rubble. These models can give better results in limit load analysis. However, post peak behaviour cannot be modelled correctly with these models as they lack strain-softening feature. Given below are the details of proposed model.

## **MATERIAL MODEL FOR ICE RUBBLE**

A continuous surface cap model (CSCM) which is proposed by Sandler et al. (1976) and further developed by Schwer and Murray (1994) is used to simulate punch through test event. Extensive calibration and validation of this model is given by and Murray, Abu-Odeh et al. (2007). The CSCM model combines the shear failure surface with hardening compaction surface smoothly and continuously by using a multiplicative formulation. The smooth intersection eliminates the numerical complexity of treating a corner region between the “failure surface” and the “cap”. Ice rubble shows softening in low to medium strain rate with

low confinement. Softening is modelled via a damage formulation. The CSCM model controls damage using a strain based energy approach. The damage formulation models both strain softening and modulus reduction. Strain softening is decrease in strength during continuous deformation after yield strength. The damage formulation is based on the work of Simo and Ju (1987). Given below is the equation for damaged stress

$$\sigma_{ij}^d = (1 - d)\sigma_{ij}^{vp} \quad (1)$$

Where  $d$  is scalar damage parameter that transforms the stress tensor without damage denoted by  $\sigma_{ij}^{vp}$ , into the stress with damage, denoted by  $\sigma_{ij}^d$ . The damage parameter “ $d$ ” ranges from zero for no damage to 1 for complete damage. Thus  $(1 - d)$  is a reduction factor whose value depends on the accumulation of damage. The effect of this reduction factor is to reduce the bulk and shear moduli isotropically (simultaneously and proportionally). A detailed theoretical description and comprehensive calibration procedure of CSCM is given in Murray (2007) and Murray, Abu-Odeh et al. (2007).

## SIMULATION OF PUNCH THROUGH TEST

In punch through tests, a platen is pushed down through a pre-cut consolidated layer forming a plug in rubble underneath. Consolidated layer underneath the platen is separated from rest of rubble field to reduce the loading capacity and separate the contribution from consolidated layer. Several punch tests were done both in-situ and in laboratories since early 1990's. After that extensive punch through tests conducted by Croasdale (1995), Timco and Cornett (1999), Heinonen and Määttänen (2000b), Azarnejad and Brown (2001), Lemee and Brown (2002) and Liferov (2005b).

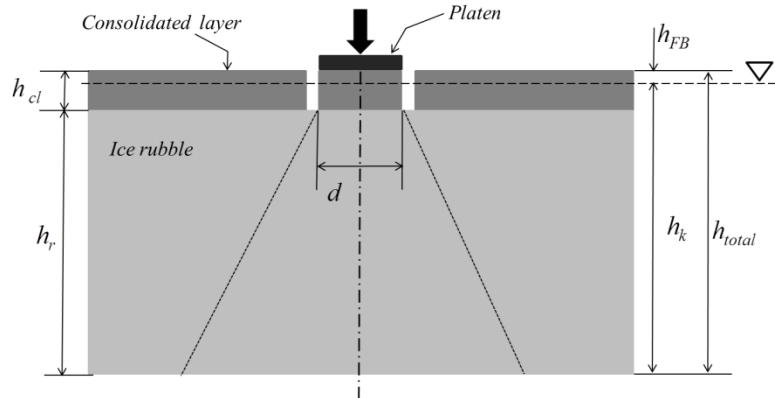


Figure 1. Principal sketch of the punch through test by Heinonen (2004)

Often rubble behaviour in punch through test in keel is approximated based on force displacement plots. There are several phenomena associated with the typical force-displacement curves. Roughly, force displacement plots can be interpreted in three parts: pre-peak, peak, post peak and residual (As shown in Figure 2). The first peak is likely to be associated to breakage of the ice rubble skeleton. The initial strength of the skeleton is assumed to be controlled by freeze bonding between ice blocks inside the rubble. After the initial peak, the behaviour of broken, loose blocks in ice rubble is mainly dominated by contact friction, interlocking and strain rate. In time history plot of chosen test, the force grows linearly with time and then it evens out and stays almost constant until failure. Heinonen (2004) reported that the load capacity of the system was almost reached and hydraulic flow was decreased due to higher internal leaks of the hydraulic system causing lower velocity. After the first peak there are subsequent peaks. So a reasonable assumption can be drawn here that cohesive type structure of ice rubble breaks at peak load and failure

propagates further as loading continues forming a plug. At end constant force is recorded which is because of buoyancy of plug.

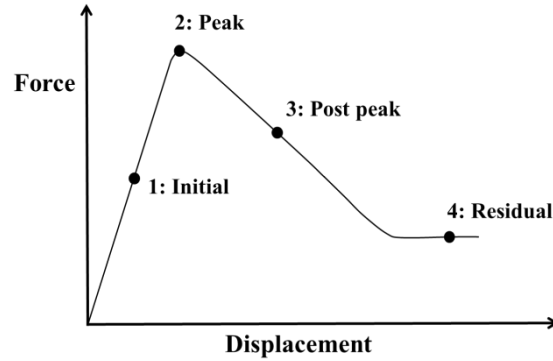


Figure 2. Typical force vs displacement graph from punch through test

To determine material model parameters a punch through test has selected from series of tests performed by Heinonen and Määtänen (2000a). This punch through test is performed in year 2000 at outside Marjaniemi, Gulf of Bothnia. To push the keel downwards cylinder-piston assembly is used. The reaction forces of piston are transmitted to nearby rubble field through approximately 9 m high mast and anchored steel wire ropes. To measure the displacement of keel rubble LVDT sensors and floating pipes were used at bottom and inside rubble. Figure 1 shows the principal sketch of the punch through test.

Table 1: Main values of the punch through test no. 0/2000 obtained from Heinonen (2004). Where  $h_k$ = keel thickness,  $h_r$ = effective rubble thickness,  $h_{cl}$ =consolidated layer thickness,  $h_{FB}$ = freeboard thickness,  $W_{cut}$ = width of cut,  $d$ = diameter of platen. All dimensions are in mm.

Test #	$h_k$	$h_r$	$h_{cl}$	$h_{FB}$	$W_{cut}$	$d$	$\eta_r$ (%)
0/2000	5200	4600	870	270	150	3000	41

The material model parameters were calibrated by comparing numerical simulation results to full-scale punch through test performed by Heinonen (2004). To simulate this chosen test an axisymmetric model is used. The assumption of asymmetry is based on the observation that the direction of platen pushing is always perpendicular to consolidated layer. The shape of platen was circular giving axisymmetric loading and boundary condition. The keel geometry was approximated to be even and assumed homogenous. The keel geometry is made long enough to be far away from loading point. Isothermal conditions were assumed. All material properties were assumed constant throughout the keel.

Rate dependent deformation mechanics were not considered in material model. Main objective of this simulation is to simulate a pre-peak, peak and post-peak behaviour of rubble correctly. Lagrangian finite element mesh formulation is used to simulate punch through test. Given below are the details of both the formulations.

### ***Lagrangian finite element mesh formulation***

An axisymmetric finite element model was created using shell element. The essential geometrical dimensions were taken from Heinonen (2004). The keel was divided into three parts namely, consolidated layer above waterline, consolidated layer below waterline and rubble. All parts were modelled with shell element with axisymmetric formulation volume weighted (LS-Dyna, ELFORM 15).

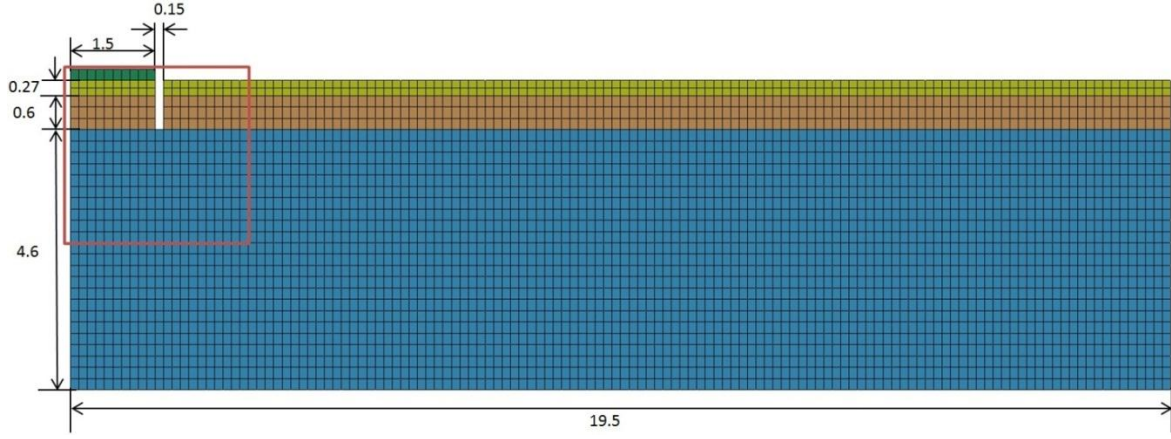


Figure 3. Axisymmetric Lagrangian mesh finite element model of Punch through test with dimensions in m.

This finite element assumes to have no spatial variation within keel geometry. Hence, all material properties are homogenous. The keel geometry was created long enough in horizontal direction to support the assumption of continuous rubble. Nodes at edge of keel geometry were constrained any displacement in horizontal and out of plane direction. The reaction forces were transmitted to nearby consolidated layer through steel ropes attached to anchors. These anchors produce uneven boundary condition in consolidated layer. So consolidated layer is fixed from anchor location to outward horizontal direction. This allows the displacement of part of consolidated layer in vertical direction. To simulate buoyancy force on keel, a finite length beam elements were created. These beam elements have one translational and one rotational degree of freedom per axis.

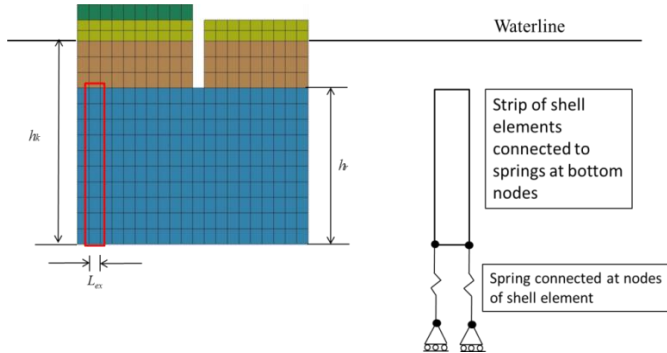


Figure 4. Illustrative sketch of beam elements employed to simulate buoyancy in finite element model

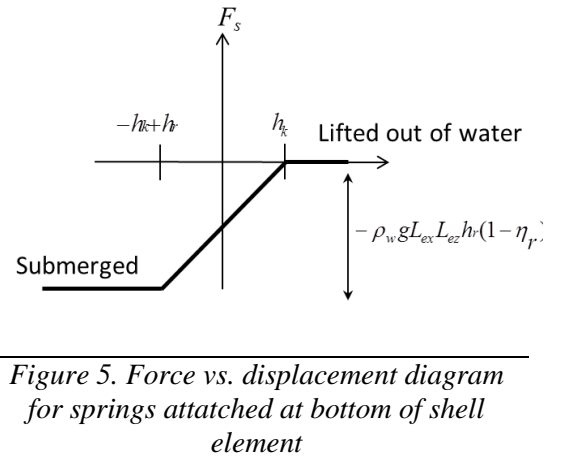


Figure 5. Force vs. displacement diagram for springs attached at bottom of shell element

Attached to bottom nodes of keel, these beam elements were assigned a translational force resulting from force displacement curve. These force displacement curves were assigned to particular set of beam elements with respect to their location in keel. Given below is the sketch illustrating the beam elements for simulating the buoyancy force. The buoyancy force on rubble is calculated as

$$F_b = \rho_w g V_s (1 - \eta_r) \quad (2)$$

where  $F_b$  is buoyancy force,  $\rho_w$  is the mass density of the water ( $1005 \text{ kg/m}^3$ ),  $g$  is gravitational acceleration ( $9.81 \text{ m/s}^2$ ),  $V_s$  is the volume of strip of rubble (shown in red colour in Figure 4) submerged in water and  $\eta_r$  is the porosity of rubble.

Additionally, translational forces in other two directions were also defined through force displacement diagram. A damping force acting as water drag is also provided through force displacement diagram for axis along the length of beam element. Given below is the formula used to calculate the water drag force

$$F_d = \frac{1}{2} V^2 C_d \rho_w A (1 - \eta_r) \quad (3)$$

where  $F_d$  is the drag force, which is by definition the force component in opposite direction of movement of object,  $\rho_w$  is the mass density of the water (1005 kg/m<sup>3</sup>),  $V$  is the velocity of the object relative to the water,  $A$  is the cross sectional area perpendicular to direction of motion and  $C_d$  is the drag coefficient – a dimensionless coefficient related to the object's geometry and taking into account both skin friction and form drag.

The platen is pushed down from per cut part of consolidated layer with linear displacement of 10 mm/s. No friction is considered between platen and consolidated layer. Elastic plastic material model is used for consolidated layer. Mesh convergence study was not performed and same mesh size used for all the rubble part. To avoid hourglass modes in under integrated shell elements, hourglass type and coefficient is added as per recommendation of LS-Dyna Hallquist (2006). Gravity is defined by using GRAVITY\_PART.

## CALIBRATION OF MATERIAL MODEL

The CSCM material model parameters were calibrated based on comparison of simulation results with chosen test data. For consolidated layer an elastic material model is used with material properties given in Table 2.

*Table 2. Parameters used in simulations for consolidated layer*

Parameter(unit)	Symbol	Value
Density (kg/m <sup>3</sup> )	$\rho_{cl}$	871
Poisons ratio	$\nu$	0.3
Elastic modulus (MPa)	$E$	8000

The density of rubble is calculated based on its porosity given by Heinonen (2004). As shown in Figure 2, typical force displacement diagram of punch through test can be divided into three parts. First part is elastic region. Until peak or yield strength, force is linear to displacement of platen. This can attribute to elastic properties of rubble. Elastic modulus was chosen based on parametric study against best fit to linear part of force displacement curve before peak. The shear modulus ( $G$ ) and bulk modulus ( $K$ ) were calculated based on relationship given in equation 1 as direct input to CSCM material model. In those relationship poisons ratio ( $\nu$ ) assumed to be 0.3. Given below are the parameters used in these simulations.

*Table 3. Yield surface parameters of CSCM*

Parameter	Symbol	Value	Parameter	Symbol	Value
Density (kg/m <sup>3</sup> )	$\rho_r$	541	Torsion surface terms	$\alpha_1$	0.737
Elastic modulus (MPa)	$E$	45		$\theta_1$	0
Shear modulus (MPa)	$G$	17.31		$\lambda_1$	0.16
Bulk modulus (MPa)	$K$	37.5		$\beta_1$	0

Triaxial compression surface terms	$\alpha$	0.016
	$\theta$	0.182
	$\lambda$	0
	$\beta$	0

Triaxial extension surface terms	$\alpha_2$	0.66
	$\theta_2$	0
	$\lambda_2$	0.16
	$\beta_2$	0

The triaxial compression parameters such as  $\alpha$  and  $\theta$  were calculated based on relationship given by Schwer and Murray (1994) to Mohr-Coulomb parameters cohesion ( $c$ ) and international friction angle ( $\phi$ ). Parametric study ensures that chosen  $\alpha$  and  $\theta$  gives approximately same peak force. Other two parameters  $\lambda$  and  $\beta$ , which represent nonlinear and exponent term of triaxial compression surface kept at 0.

Table 4. Cap hardening parameters of CSCM

Parameter	Symbol	Value
Cap ellipticity ratio	$R$	9.44
Initial intercept of the cap surface	$X_D$	0.595
The maximum plastic volumetric strain	$W$	0.05
The linear shape parameters	$D_1$	0.001
The quadratic shape parameters	$D_2$	0.65

To define cap-hardening laws five input parameters ( $X_D$ ,  $W$ ,  $D_1$ ,  $D_2$ , and  $R$ ) are selected from parametric study where simulated force displacement curve compared with modified test curve. Bottom displacement also compared.

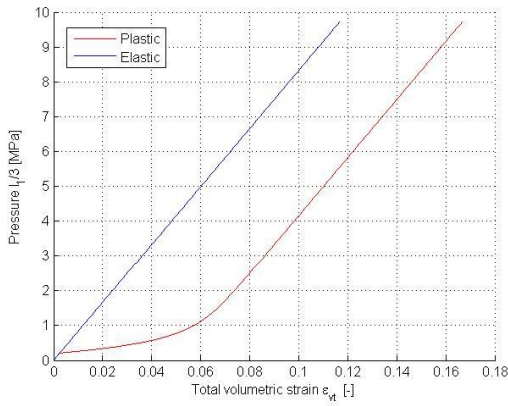


Figure 6. Plot of first invariant of stress tensor  $I_1$  versus plastic volumetric strain  $\varepsilon_v^p$  for chosen value of  $X_0$ ,  $W$ ,  $D_1$ ,  $D_2$ , and  $R$

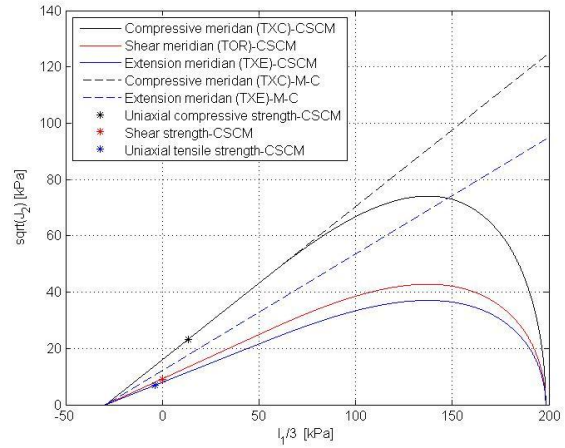


Figure 7. 2D yield surface plotting of CSCM criterion and Mohr-Coulomb criterion fitted to data for ice rubble

Softening part mainly controlled by Damage parameters. The CSCM model handles damage using a strain-based energy approach. When this energy exceeds a material damage threshold, damage is initiated and accumulated via the parameter  $D$  (refer equation 1). The damage threshold is determined using two different formulations for brittle and ductile damage. Brittle damage accumulates in the CSCM model only when the pressure is tensile. Ductile damage, on the other hand, accumulates when the pressure is compressive. Given below are the values for selected parameters.



Table 5. Damage parameters of CSCM

Parameter	Symbol	Value
Ductile shape softening parameter	$B$	20
Fracture energy in uniaxial compression ( $\text{J/m}^2$ )	$G_{fc}$	0.4
Brittle shape softening parameter	$D$	1
Fracture energy in uniaxial tension ( $\text{J/m}^2$ )	$G_{fs}$	0.065
Fracture energy in pure shear ( $\text{J/m}^2$ )	$G_{ft}$	0.065

A 2D yield surface plotted with chosen parameters for CSCM material model. Figure 7 shows 2D yield surface plot of the model using chosen parameters. In this simulation damage parameters were selected based on fit to post peak part of experimental force displacement plot.

## RESULTS ANALYSIS

Results are analysed based on failure modes described earlier. As platen moves down, the forces on platen increased with high rate and reached peak value for relatively small displacement. From simulation point of view this can be seen as failure of freeze bonding of ice blocks and peak value is direct indication of breaking those bonds.

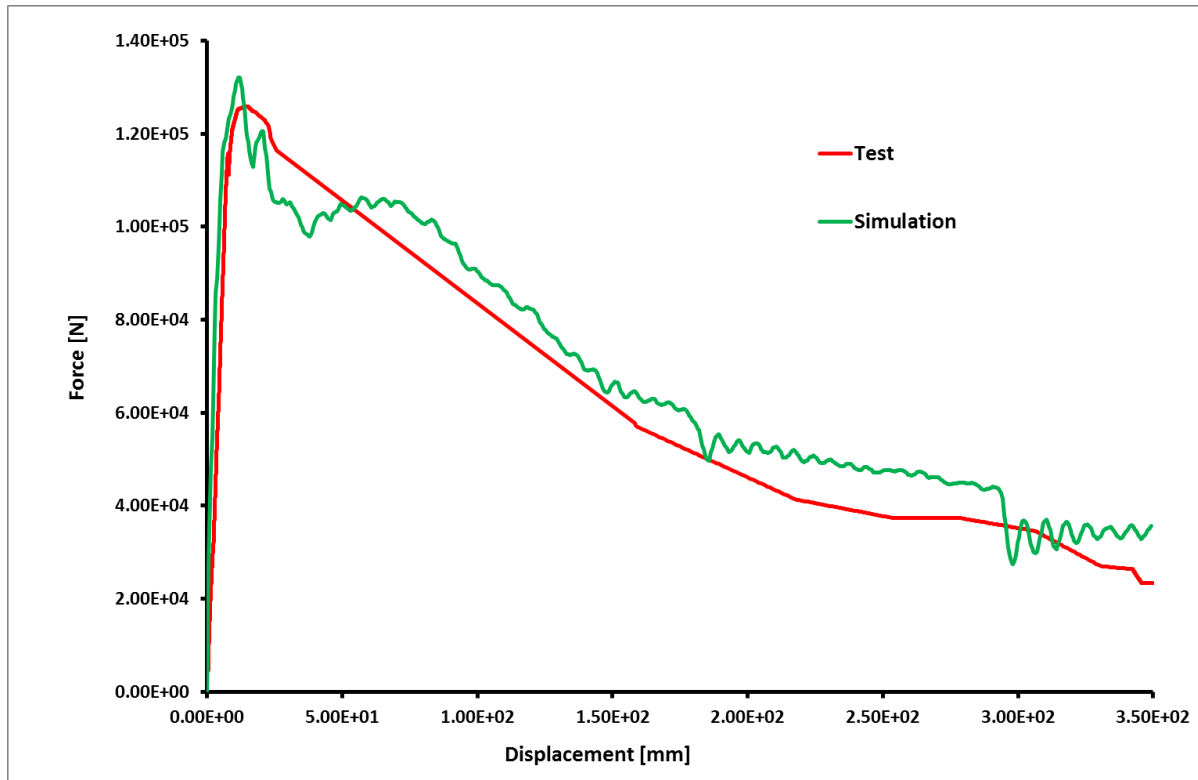


Figure 8. Force displacement diagram for test # 0/2000 compared with simulated with Lagrangian finite element.

In Figure 8 comparison of test to simulation is shown. As the peak force was seen clearly in actual force displacement plot, assumed peak from modified force displacement plot matches with simulated peak force. Internal friction angle and cohesion are adjusted to match the peak force. Also young's modulus was chosen to fit the slope of initial loading phase in force



displacement diagram. During initial phases of platen loading, failure progress downwards forms a plug. Field test results shows outward growing of a plug. In simulation also an outward growing plug forms but dimensions differs. This can be explained by continuum elements used in finite elements. The failure mode is determined by the stress state characteristic, which depends on both the loading and boundary conditions and on the keel geometry as well. The keel is supported by buoyancy force, due to its porous nature. During vertical loading keel fails by shearing and compression.

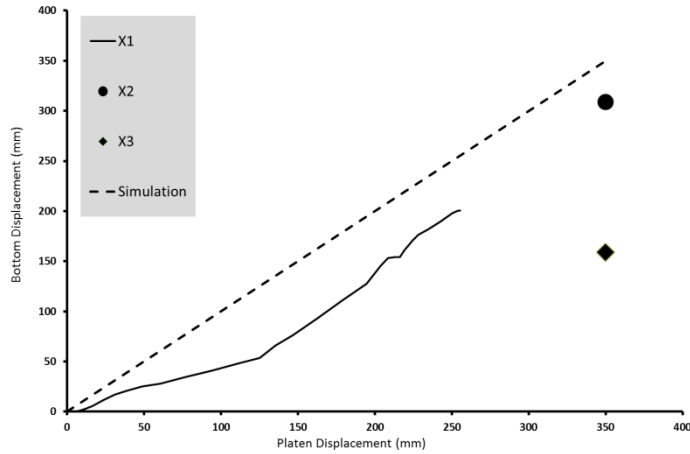


Figure 9. Bottom displacement of keel recorded by different sensors plotted against platen displacement.

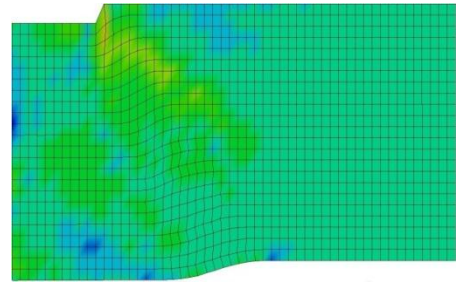
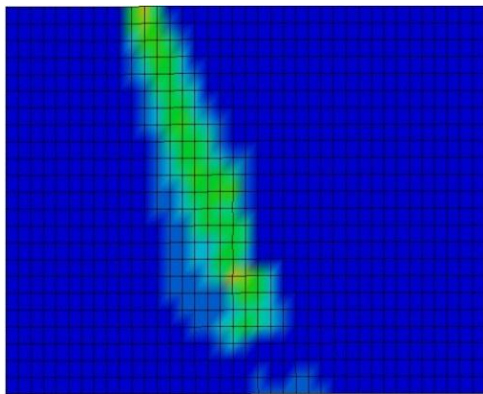
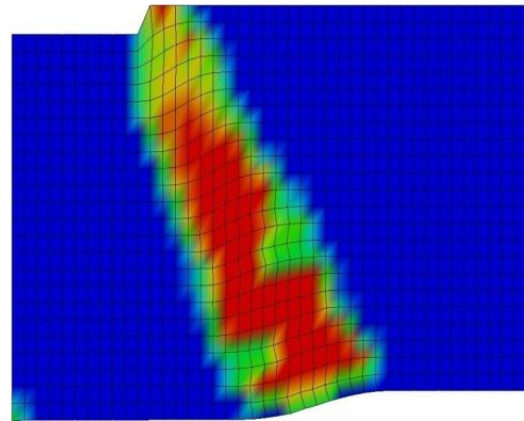


Figure 10. Stress distribution in XY plane in finite element model at 350 mm displacement of platen

Figure 9 shows bottom displacement of keel. Simulation predicts constantly increasing displacement of X1 sensor, whereas a test shows somewhat lower displacement of corresponding sensor. Ductile damage is dominating in the simulation. Ductile damage occurs when the mean stress in compressive.



(a)



(b)

Figure 11. Damage progression at (a) peak force and (b) at 350 mm displacement of platen

Figure 11 it is clearly seen that damage starts at the edge of platen and progress towards the bottom. At 350 mm displacement of platen, elements in shear zone show the maximum damage.

## DISCUSSION AND CONCLUSIONS

In total, 22 parameters were needed to define the proposed continuous surface cap model. However, some approximations and simplification can reduce that number to 15. Material parameters were calibrated based on response to the measured force displacement diagram. In the field tests load and deformation after peak load was insufficiently measured and therefore modelled post peak behaviour is somewhat uncertain.

A 2D surface plotted for CSCM in compression, shear and extension meridian to ensure the validity of chosen values of material parameters. Those parameters are also plotted for Mohr-Coulomb in compression and tension. An axisymmetric model with plane strain assumption gives reasonably good result. Although to get the clear view of rubble deformation 3D model is required. The displacement nodes at the bottom of keel were smaller than corresponding points in rubble obtained by sensors X1, X2 and X3, see Figure 9.

The proposed finite element model simulates initial, peak and post peak behaviour. The simulation is in good agreement with the full-scale punch through test. In this simulation as for most other rubble simulations ductile damage was dominating over brittle damage.

Since rate effect was not considered, strength surface remains constant in CSCM model.

## ACKNOWLEDGEMENTS

This work was supported by the Research Council of Norway, project number 195153.

## REFERENCE

- Azarnejad, A. and Brown, T. G. 2001. "Ice rubble behaviour in punch tests." *Journal of cold regions engineering* 15(3): 135-153.
- Azarnejad, A., et al., 1999. Ice rubble strength from small scale punch through tests. *Proceedings of OMAE99, 19th International Conference on Offshore Mechanics and Arctic Engineering*.
- Croasdale, K. R., 1995. In-situ testing of ice rubble properties. . Report submitted to The National Energy Board, Canada, . Calgary, Alberta.
- Fransson, L., 1985. Load bearing capacity of an ice cover subjected to concentrated loads. *Proceedings of the 4th International Offshore Mechanics and Arctic Engineering Symposium: presented at Energy-Sources Technology Conference & Exhibition, Dallas, Texas*.
- Hallquist, J. O., 2006. *LS-Dyna Theoretical Manual*. Livermore, Ca, Livermore Software Technology Corporation.
- Heinonen, J., 2004. Constitutive modeling of ice rubble in first-year ridge keel. VTT Technical Research Centre of Finland. Publications. Espoo, Finland, . PhD Thesis: 3-142.
- Heinonen, J. and Mänttinen, M., 2000a. LOLEIF ridge-loading experiments-analysis of rubble strength in ridge keel punch test. *Proceedings of the 15th International Symposium on Ice*.
- Heinonen, J. and Mänttinen, M., 2000b. LOLEIF ridge-loading experiments-analysis of rubble strength in ridge keel punch test. *Proc. of the 15th International Symposium on Ice (IAHR)*, Gdansk, Poland.
- Høyland, K. V., 2002. "Consolidation of first-year sea ice ridges." *Journal of geophysical research* 107(C6): 3062.
- Høyland, K. V., et al., 2004. Mechanical properties of ice ridges and level ice, in-situ and laboratory testing In *Proc. of the 17th Int. Symp. on Ice (IAHR)*. St. Petersburg, Russia. Vol. 1, pp. 69-75.
- Lemee, E. and Brown, T., 2002. Small-scale plane strain punch tests. *Ice in the Environment: Proceedings of the 16th IAHR International Symposium on Ice*.
- Liferov, P., 2005a. First-year ice ridge scour and some aspects of ice rubble behaviour. Trondheim, Norway, Norwegian University of Science and Technology, Faculty of Engineering Science and Technology, Department of Civil and Transport Engineering. PhD Thesis.

Liferov, P., 2005b. "Ice rubble behaviour and strength: Part II. Modelling." *Cold Regions Science and Technology* 41(2): 153-163.

Liferov, P., Bonnemaire, B., 2005c. "Ice rubble behaviour and strength: Part I. Review of testing and interpretation of results." *Cold Regions Science and Technology* 41(2): 135-151.

Murray, Y. D., 2007. Users manual for LS-DYNA concrete material model 159.

Murray, Y. D., A. Y. Abu-Odeh, et al., 2007. Evaluation of LS-DYNA concrete material model 159.

Sandler, I. S., et al., 1976. "Generalized cap model for geological materials." *Journal of the Geotechnical Engineering Division* 102(7): 683-699.

Schwer, L. E. and Murray, Y. D., 1994. "A three-invariant smooth cap model with mixed hardening." *International Journal for Numerical and Analytical Methods in Geomechanics* 18(10): 657-688.

Serré, N., 2011. Study of the rubble ice action in scale-model ice ridge impact on seabed structures. Faculty of Engineering Science and Technology, Department of Civil and Transport Engineering. NTNU, Trondheim, Norway, Norwegian University of Science and Technology. PhD in Civil and Transport Engineering.

Shafrova, S., 2007. First-year sea ice features. Investigation of ice field strength heterogeneity and modelling of ice rubble behaviour. Department of Civil and Transport Engineering, Faculty of Engineering Science and Technology. Trondheim, August 2007, Norwegian University of Science and Technology. The degree of philosophiae doctor: 170.

Simo, J. and Ju, J., 1987. "Strain-and stress-based continuum damage models—II. Computational aspects." *International journal of solids and structures* 23(7): 841-869.

Timco, G. and Cornett, A., 1999. Is  $\phi$  a constant for broken ice rubble. 10th workshop on river ice , river ice management with a changing climate: dealing with extreme events, J.C.Doering Ed.

Timco, G., et al., 2000. An overview of first-year sea ice ridges. Technical Report: 154.

Wong, T. T., et al., 1990. "A constitutive model for broken ice." *Cold Regions Science and Technology* 17(3): 241-252.